CUTFEM: DISCRETIZING GEOMETRY AND PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

Many advanced engineering problems require the numerical solution of multidomain, multidimension, multiphysics and multimaterial problems with interfaces. When the interface geometry is highly complex or evolving in time, the generation of conforming meshes may become prohibitively expensive, thereby severely limiting the scope of conventional discretization methods.

For instance, the simulation of blood flow dynamics in vessel geometries requires a series of highly non-trivial steps to generate a high quality, full 3D finite element mesh from biomedical image data. Similar challenging and computationally costly preprocessing steps are required to transform geological image data into conforming domain discretizations which respect complex structures such as faults and large scale networks of fractures. Even if an initial mesh is provided, the geometry of the model domain might change substantially in the course of the simulation, as in, e.g., fluid-structure interaction and free surface flow problems, rendering even recent algorithms for moving meshes infeasible. Similar challenges arise in more elaborated optimization problems, e.g. when the shape of the problem domain is subject to the optimization process and the optimization procedure must solve a series of forward problems for different geometric configuration.

In this talk, we focus on recent finite element methods on cut meshes (CutFEM) as one possible remedy. CutFEM technologies allow flexible representations of complex or rapidly changing geometries by decomposing the computational domain into several, possibly overlapping domains. Alternatively, complex geometries only described by some surface representation can easily be embedded into a structured background mesh. In the first part of this talk, we briefly review how finite element schemes on cut and composite meshes can be designed by using Nitsche-type imposition of interface and boundary conditions. To make the formulations robust, optimally convergent and to avoid ill-conditioned linear algebra systems, so-called ghost-penalties are added in the vicinity of the boundary and interface. In the second part we demonstrate how CutFEM techniques can be employed to address various challenges from mesh generation to fluid-structure interaction problems, solving PDE systems on embedded manifolds of arbitrary co-dimension and PDE systems posed on and coupled through domains of different topological dimensionality.