

Régis de la Brèche

Gál's sums and applications

We evaluate the asymptotic size of various sums of Gál type, in particular

$$S(\mathcal{M}) := \sum_{m,n \in \mathcal{M}} \sqrt{\frac{(m,n)}{[m,n]}}$$

where \mathcal{M} is a finite set of integers. Elaborating on methods recently developed by Bondarenko and Seip, we obtain an asymptotic formula for

$$\log \left(\sup_{|\mathcal{M}|=N} S(\mathcal{M})/N \right)$$

and derive new lower bounds for localized extreme values of the Riemann zeta-function, for extremal values of some Dirichlet L -functions at $s = 1/2$, and for large character sums.