The $n \times n$ Redheffer matrix [1977]

$$A_n = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & \cdots \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & \cdots \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots
\end{pmatrix}$$

has some very interesting properties, yet very little progress has been made in the last 30 years and several fundamental questions remain unresolved. We give an overview of some of them and outlines of some proofs.