We describe the spectra and localization properties of the N-site banded one-dimensional highly asymmetric random matrices that arise naturally in sparse neural networks, and also in deep learning models with tunable feed-forward interaction strengths. When N is large, approximately equal numbers of random excitatory and inhibitory connections lead to spatially localized eigenfunctions, and an intricate eigenvalue spectrum in the complex plane that controls the spontaneous activity and induced response. A finite fraction of the eigenvalues condense onto the real or imaginary axes, with remarkable spectral symmetries and a strongly diverging localization length near the origin. When chains with periodic boundary conditions become directed, with a systematic directional bias superimposed on the randomness, a hole centered on the origin opens up in the density-of-states in the complex plane. All states are extended on the rim of this hole, while the localized eigenvalues outside the hole are unchanged. Similar results are obtained for more realistic neural networks that obey "Dale's Law" (each site is purely excitatory or inhibitory). Related problems arise in random ecological networks and in chains of artificial cells with randomly coupled gene expression patterns.